On Block Codes Constructed by Tail Biting Unit Memory Trellis Codes

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1 Introduction

There are three methods to obtain block codes from trellis codes, i.e., those of (1) Tail Termination (TT), (2) Direct Truncation (DT), and (3) Tail Biting (TB) [2],[5]. In this paper, we discuss the exponential error bounds for block codes constructed by unit memory (UM) trellis codes [4] based on the TB method. Throughout this paper, we assume the channel be discrete memoryless with channel capacity C.

2 Unit Memory (UM) Trellis Codes

For the conventional trellis code of branch constraint length v, letting $v = 1$ and introducing an (n_0, k_0) block code to each branch as a component code, we have a unit memory (UM) trellis code [4],[6]. Let a (u, n_0, k_0) UM trellis code over GF(q) be a code of branch length u (≥ 2), length of component code n_0, number of information symbols of component code k_0, and rate $r = (k_0/n_0)\ln q$ [nats/symbol], then we have the following lemma.

Lemma 1 ([6]) *There exists a (u, n_0, k_0) unit memory (UM) trellis code for which the probability of decoding error* $\Pr(\mathcal{E})$ *satisfies:*

$$\Pr(\mathcal{E}) \leq \exp[-n_0 e_{UM}(r)] \quad (0 \leq r < C) \tag{2.1}$$

$$e_{UM}(r) = \begin{cases} 2E(r/2) & (0 \leq r \leq R_{\text{comp}}) \\ e(r) & (R_{\text{comp}} < r < C) \end{cases}. \tag{2.2}$$

□

3 Tail Biting (TB) UM trellis codes

Consider a (u_0, n_0, k_0) UM trellis code. Then we have block codes of length $N = u_0 n_0$ with no loss in rates constructed by tail biting (TB) techniques [5]. We have the following theorem.

Theorem 1 *There exists a (u_0, n_0, k_0) tail biting unit memory (TB-UM) trellis code for which the probability of decoding error satisfies:*

$$\Pr(\mathcal{E}) \leq \exp[-N E_{TB\text{-}UM}(r)] \quad (0 \leq r < C) \tag{3.1}$$

$$E_{TB\text{-}UM}(R) = \min_{\theta_0 = 1/u_0, u_0 = 2,3,4,\cdots} \{\theta_0 e_{UM}(r), E[(1-\theta_0)r]\} \tag{3.2}$$

where $N = u_0 n_0$ and $r = (k_0/n_0)\ln q$. □

Example 1 *Over a very noisy channel (VNC), error exponents $E_{TB\text{-}UM}(r)$ for TB-UM trellis codes are depicted in Figure 1 together with those $E_{TT\text{-}UM}(r)$ [1],[2] and $E_{DT\text{-}UM}(r)$ [2] for TT-UM and DT-UM trellis codes respectively, although the detailed discussions [3] are omitted here.*

We see that for a given u_0, $E_{TT\text{-}UM}(r) \leq E_{TB\text{-}UM}(r)$ holds for all rates. Since $u_0 = 2, 3, \cdots$, $E_{TT\text{-}UM}(R)$ and $E_{DT\text{-}UM}(r)$ are given by line graphs and they are close to the curve $E(R)$ as its tangent lines, $E_{TT\text{-}UM}(R) \leq E(R)$ always holds. Note that as the result of numerical computation, the second term of rhs of (3.2) does not affect $E_{TB\text{-}UM}(r)$ for all rates over the VNC. □

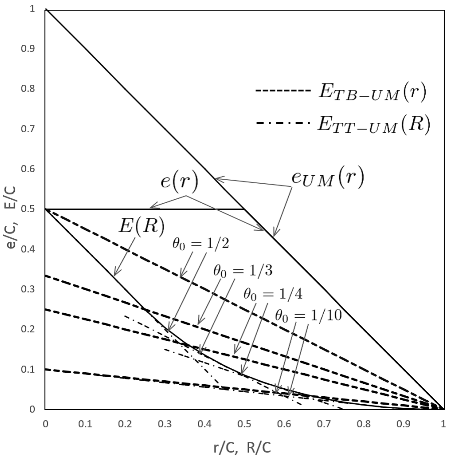

Figure 1 Error exponents for a very noisy channel.

4 Concluding Remarks

We have shown that the error exponent of block code can be improved by using the tail biting unit memory (TB-UM) trellis code. Discussion on decoding complexity of the TB-UM code is remained as a further research.

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