Exponential Error Bounds for Tail Biting Trellis Codes

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1 Introduction

Tail biting (TB) trellis codes [4] are known to be one of the most powerful codes for converting trellis codes into block codes with no loss in rates. These codes are practically applied to the LTE in 4G systems. In this paper, we discuss the performance of them from random coding arguments. Since we require complete maximum-likelihood decoding (MLD), the relationship between the probability of decoding error $\Pr(\mathcal{E})$ and the decoding complexity $G(N)$ at given rates r, or R, can be made clear [3], where N is the code length. Throughout this paper, we assume that the channel be discrete memoryless with channel capacity C.

2 Trellis Codes

Let a (u, v, b) (conventional) trellis code over GF(q) be a code of branch length u, branch constraint length v, yielding b channel symbols per branch, where the rate $r = (1/b)\ln q$ [nats/symbol].

Lemma 1 ([1]) *The probability of decoding error* $\Pr(\mathcal{E})$ *and the decoding complexity* $G(v)$ *for a* (u, v, b) *trellis code satisfy:*

$$\Pr(\mathcal{E}) \le uK_1 \exp[-vbE_0(\rho)] \quad (0 \le \rho \le 1) \quad (2.1)$$

$$\le \exp\{-vb[e(r) - o(1)]\} \quad (0 \le r < C), \quad (2.2)$$

$$G(v) \sim u^2 q^v \quad (2.3)$$

where K_1 *is a constant independent of* v, $o(1) \to 0$ $(v \to \infty)$, *and* $e(\cdot)$ *is (a lower bound on) the trellis code exponent [1] given by*

$$e(r) = \begin{cases} E_0(1) & 0 \le r \le R_{\mathrm{comp}} \\ E_0(\rho_r) & R_{\mathrm{comp}} < r = E_0(\rho_r)/\rho_r < C \end{cases} \quad (2.4)$$

and $E_0(\cdot)$ *is Gallager's function, and* $R_{\mathrm{comp}} = E_0(1)$ *is the computational cut-off rate of the channel.* □

Note that the following relation holds between $E(R)$ and $e(r)$:

$$E(R) = \max_{0<\mu\le 1} (1-\mu)e(R/\mu) \quad (2.5)$$

which is called the concatenation construction [1].

3 Tail biting Trellis Codes

Consider a (u_0, v, b) trellis code. Then, we have a $(u_0 b, u_0)$ block code over GF(q) by the tail biting method, where $N = u_0 b, \theta_0 = v/u_0$, and $r = (1/b)\ln q$.

Theorem 1 *For any* θ_0 $(0 \le \theta_0 \le 1)$, *there exists a* (u_0, v, b) *tail biting trellis code for which the probability of decoding error* $\Pr(\mathcal{E})$ *and the decoding complexity* $G(N)$ *satisfy:*

$$\Pr(\mathcal{E}) \le \exp[-NE_{\mathrm{TB}}(r)], \quad (3.1)$$

$$G(N) \sim N^2 q^{2v} = N^2 \exp[2N\theta r] \quad (3.2)$$

where

$$E_{\mathrm{TB}}(r) = \theta_0 e(r) - o(1) \quad (0 \le \theta_0 \le 1, 0 \le r < C), \quad (3.3)$$

$o(1) \to 0$ $(N \to \infty)$, *and* $N = u_0 b$. □

Example 1 *Over a very noisy channel (VNC), error exponents* $E_{\mathrm{TB}}(r)$ *for TB trellis codes are depicted in Figure 1 together with those* $e(r)$ *and* $E_{\mathrm{TT}}(R)$ *for tail termination trellis codes [1], where* $R = (1-\theta_0)r$. *We see that for a given* $\theta_0, E_{\mathrm{TT}}(R) \le E_{TB}(r)$ *holds for all rates* $r = R$ *over VNC* □

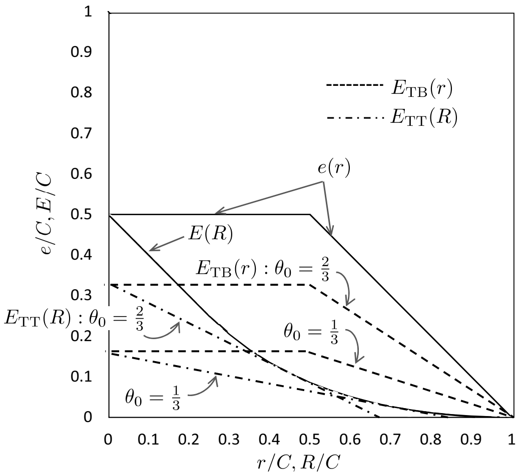

Figure 1 Error exponents for a very noisy channel..

4 Concluding Remarks

We have shown that the error exponent of block code can be improved by using the TB trellis code. Although the detailed discussions are omitted here, there is a possibility such that at some rates larger error exponents can suppress the growth of decoder complexity [2].

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